Bachelor of Science (B.Sc.) Semester-III (C.B.S.) Examination

MATHEMATICS

(Differential Equations and Group Homomorphism)

Paper—II

Time—Three Hours] [Maximum Marks—60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) If λ_j and λ_k are roots of the equation $J_n(\lambda a) = 0$, then prove that :

$$\int_0^a xJ_n(\lambda_j x)\,J_n(\lambda_k x)\,dx = 0 \text{ if } j \neq k.$$ 6

(B) Prove that : 6

(i) $J_{-1/2}(x) = \sqrt{(2/\pi x)}\cos x$

(ii) $J_{1/2}(x) = \sqrt{(2/\pi x)}\cdot\sin x$

(iii) $(J_{1/2}(x))^2 + (J_{-1/2}(x))^2 = 2/\pi x$

OR

(C) Prove that :

$$P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n} (x^2 - 1)^n .$$ 6

(D) Prove that :

$$\int_{-1}^{1} x^n \rho_n(x)\, dx = \frac{2^{n+1}(n!)^2}{(2n+1)!} .$$ 6

UNIT—II

2. (A) Let f(t) and g(t) be continuous for t > 0, then prove that :

$$L[a\, f(t) + bg(t)] = a\, L[f(t)] + b\, L[g(t)]$$

where a and b are constants. Hence find the Laplace transform of $f(t) = (3e^{2t} - 4)^2$. 6

(B) Find :

$$L^{-1}\left[\log\left(1 + \frac{1}{s^2}\right)\right].$$ 6

OR

(C) Let L[F(t)] = F(s), then prove that $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$, provided the limits exist. Verify this result for the function $f(t) = e^{-2t}$. 6

(D) Find the inverse Laplace transform of :

$$\frac{s}{(s+1)^2 (s^2+1)} .$$ 6

UNIT—III

3\. (A) Solve $y''' + 2y'' - y' - 2y = 0$, given that $y(0) = y'(0) = 0$ and $y''(0) = 6$ by method of Laplace transform, where $y = y(t)$. 6

(B) Solve $3\frac{dx}{dt}+\frac{dy}{dt}+2x=1$, $\frac{dx}{dt}+4\frac{dy}{dt}+3y=0$, given $x = 3$, $y = 0$ when $t = 0$, where $x = x(t)$, $y = y(t)$. 6

OR

(C) Solve $y'' + ty' - y = 0$, given that $y(0) = 0$, $y'(0) = 2$, where $y = y(t)$. 6

(D) Find the Fourier sine transform of $\frac{e^{-\lambda x}}{x}$, $x > 0$. 6

UNIT—IV

4\. (A) Prove that the set of all cosets of a normal subgroup of a group G is a group under the composition of coset-multiplication. 6

(B) Prove that the intersection of two normal subgroups of a group is a normal subgroup. 6

OR

(C) Let f be a homomorphism of a group G onto a group G' with kernel K and 'a' be a given element of G such that $f(a) = a' \in G'$. Then prove that the set of all those elements of G which have their image a' in G' is the coset Ka of K in G. 6

(D) Let G be the multiplicative group of all positive real numbers and G', the additive group of real numbers. Show that the mapping $g : G \rightarrow G'$ defined by $g(x) = \log x$, $x \in G$ is isomorphic. Also find the kernel of g. 6

UNIT—V

5. (A) Prove that :

$$J_o(x)=1-\frac{x^2}{2^2}+\frac{x^4}{2^2\cdot 4^2}-\frac{x^6}{2^2\cdot 4^2\cdot 6^2}+\ldots\ldots.. \quad 1½$$

(B) Prove that :

$$x^2=\frac{1}{3}P_o(x)+\frac{2}{3}P_2(x). \quad 1½$$

(C) Prove that :

$$L(t^n)=\frac{n!}{s^{n+1}};n=0,1,2,3,\ldots\ldots. \quad 1½$$

(D) Find :

$$L^{-1}\left[\frac{1}{s^2-4s+20}\right]. \quad 1½$$

(E) Let u(x, t) be a function defined for t > 0 and $x \in [a, b]$. Show that $L\left(\frac{\partial u}{\partial x}\right)=\frac{dU}{dx}$, where $U = U(x, s) = L\{u(x, t)\}$. 1½

(F) Find L(x) for $\frac{dx}{dt}+x=\sin \omega t$,, x(0) = 2. 1½

(G) How many elements of the cyclic group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ can be used as the generators of G ? 1½

(H) Prove that every subgroup of an abelian group is normal. 1½